Combinatorial Analysis of Double Rook Attack Patterns for Checkmate in Chess Midgame Based on Grid Position and Symmetry

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***Abstract*—Chess is a strategy board sport played by two people on a square board. The game of chess has many possible strategies that can be analyzed mathematically, especially in the mid-game phase to the end of the game, when some pieces have been active on the board and when some pieces are dead attacked by the opponent's pieces. One of the attack combinations that are often used in chess is the strategy between two rooks to put pressure on the opponent's piece, especially the position of the opponent's king. This study aims to examine various patterns of placement of two rooks that can produce potential dangerous attacks that are able to create checkmate conditions. The approach used in this study is based on combinatorics, taking into account the position of the two rooks on the 8×8 chessboard, as well as considering aspects of symmetry and control of the rows. This analysis research was carried out by calculating the possibility of placing two rooks that do not obstruct each other and are able to control strategic areas on the chessboard. This research is expected to provide additional insights into the application of combinatorial concepts, symmetric patterns, and grid representation.**

Keywords—component; chess; rook; combinatorics; symmetry; grid; Attack strategy.

I. Introduction

In a chess game, each piece has a unique move. The rook is one of the pieces with the widest and most flexible movement horizontally and vertically, as long as it is not blocked by other pieces which makes it one of the pieces used in the formation of attacks in the middle phase of the game, when the position of the board is relatively more open.

The Rook plays a crucial offensive role as soon as the path before him opened. Compared to other chess pieces, rooks are able to quickly gain control over open files and ranks, which are often the main pillars of coordinated attacks. In various attack strategies, rooks are often used as pawns that can fully control a line, confine the opponent's king's position, and even create deadly checkmates.

In professional-level matches, an attack pattern involving two rooks is often one of the most complex pressures for an opponent to anticipate. Strategies such as double rook battery, file domination, and seventh-rank pressure show the power of coordination between the two rooks that can dominate the course of the match.

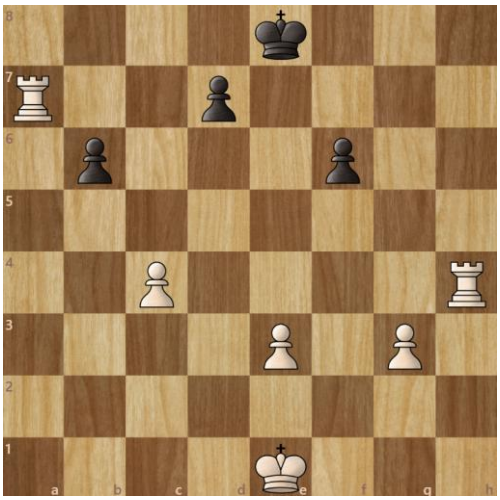

Fig 1.1 Illustration of the coordination of two rooks in an attack. (Taken from chess.com)

This illustration raises an interesting mathematical problem related to the many possibilities of placing two rooks on a chessboard measuring 8×8. Combinatorial rules became relevant in calculating all the possibilities of various attack patterns that could arise from the various positions of the rooks. In addition, the symmetry of the chessboard can affect the efficiency of the placement combination, due to the repetition of the pattern due to this symmetry. The combinatorial analysis of two rooks has an important role in creating a tighter control area and making it difficult for opponents to move. On the other hand, if the placement of the two rooks is placed far apart, it will reduce the control of the area. Thus, in addition to the large number of combinations that can be formed, the effectiveness of each movement of the two rooks in the formation of attacks is an important aspect of this study.

II. Theoretical Foundations

A. Chess Board Grid System

The chess board consists of 64 tiles arranged in the form of a grid measuring 8×8. Each tile can be represented by a coordinate that indicates its position on a specific row. Mathematically, this position is expressed as an ordered pair of

a set of integers between 1 and 8. The set of tile positions in a chessboard can be written as follows:

$$P = \{(x, y) \mid x, y \in \{1, 2, 3, 4, 5, 6, 7, 8\}\}$$

So that the total number of tiles on the chessboard can be calculated by multiplying the number of rows by the number of columns, namely:

$$P = \{(x, y) \mid 1 \leq x \leq 8, 1 \leq y \leq 8\}$$

Total number of tiles:

$$|P| = 8 \times 8 = 64$$

Rank/File	a	b	c	d	e	f	g	h
8	[0,7] a8	[1,7] b8	[2,7] c8	[3,7] d8	[4,7] e8	[5,7] f8	[6,7] g8	[7,7] h8
7	[0,6] a7	[1,6] b7	[2,6] c7	[3,6] d7	[4,6] e7	[5,6] f7	[6,6] g7	[7,6] h7
6	[0,5] a6	[1,5] b6	[2,5] c6	[3,5] d6	[4,5] e6	[5,5] f6	[6,5] g6	[7,5] h6
5	[0,4] a5	[1,4] b5	[2,4] c5	[3,4] d5	[4,4] e5	[5,4] f5	[6,4] g5	[7,4] h5
4	[0,3] a4	[1,3] b4	[2,3] c4	[3,3] d4	[4,3] e4	[5,3] f4	[6,3] g4	[7,3] h4
3	[0,2] a3	[1,2] b3	[2,2] c3	[3,2] d3	[4,2] e3	[5,2] f3	[6,2] g3	[7,2] h3
2	[0,1] a2	[1,1] b2	[2,1] c2	[3,1] d2	[4,1] e2	[5,1] f2	[6,1] g2	[7,1] h2
1	[0,0] a1	[1,0] b1	[2,0] c1	[3,0] d1	[4,0] e1	[5,0] f1	[6,0] g1	[7,0] h1

Fig 2.1 Matrix representation

The Chess Board can be represented as an 8×8 matrix where each element represents a tile. This matrix representation can make position analysis easier.

B. *Movement of Rooks in a Chessboard*

The rook has unlimited straight movement horizontally or vertically, as long as it is not obstructed by other pieces. In this analysis, the focus is on the position of placing two rooks simultaneously, with the following conditions:

1. Horizontal movement position
   The rook can move from position (x, y) to (x, z) on condition z ≠ y
2. Vertical movement position
   rook can be moved from position (x, y) to (z, y) provided z ≠ x

Thus, in developing an optimal strategy for the placement of the position of two rooks, maximum coordination is needed without interference with each other, and can provide complex pressure to the opponent, so that the opponent's defense can be easily penetrated.

C. *Combinatorial Concept of Two Rooks Placement*

In the initial concept, the placement of two rooks can be represented using the concept of combination. Because, there are 64 tiles available that will be selected by 2 different tiles to place the position of two rooks, so the number of combinations that may be formed are:

$$C_{total} = C(64, 2) = \frac{64!}{2! \times (64 - 2)!} = 2016$$

This result is an entire pair of tiles that can be randomly selected by the rook without considering the relative position of the two rooks on the chessboard. However, from 2016 this possibility is not all that can qualify as an effective placement.

This study will calculate all the possibilities of positioning two conditional rook, namely two rook must not be in the same row or column. This limitation is very important, as the position of two rooks if placed in the same row or column will reduce the overall effectiveness of the chessboard's space control.

Thus, a more specific combinatorial concept is needed to calculate all the possible placement of two valid rooks. Therefore, an enumeration approach is needed that not only avoids the repetition of positions, but also limits the placement of the positions of two rooks that collide with each other.

D. *Rule of Product Approach*

In the rule of product approach, only focus on the process of placing two rooks that will be used to help control the space on the chessboard. This rule of product approach will be divided into two stages as follows:

1. Placement of the first rook
   The initial condition is that all the tiles on the chessboard are empty and no other pieces are placed on the chessboard. Since all the tiles on the chessboard are empty, we have 64 placement options:

   $$P_1 = 64$$

2. Placement of the second rook on a plot that is not one row and one column with the first rook
   The condition is that if the first rook is placed in a certain square, then there will be 7 other tiles in the row and 7 other tiles in the column that do not collide with the position of the first rook. Thus, the total forbidden plot for the second rook is:

   $$L = 7\,(row) + 7\,(column) = 14$$

   Since 14 plots are prohibited from being used by the second rook, the remaining legal plots that can be used by the second rook are:

   $$P_2 = 64 - 1 - 14 = 49$$

   The total number of ways to place two rooks without obstructing each other are:

   $$C_{totalValid} = P_1 \times P_2 = 64 \times 49 = 3136$$

   However, since the order of placement is not taken into account (the first rook in A and the second rook in B is the same as the first rook in B and the second rook in A), it is necessary to divide the result by 2:

   $$C_{fix} = \frac{3136}{2} = 1568$$

   So, there are 1568 chances that two rooks can be legally placed on an 8×8 chess board without getting in each

other's way. These results will be the initial basis for the combinatorial analysis in this study.

E. Aspects of Symmetry in a Chessboard

The chessboard has a symmetrical nature, so some of the placement of the position of the rook is considered strategically. Symmetrical in question includes:

1. Vertical reflection (mirror from right to left)
2. Horizontal reflection (mirror from top to bottom)
3. Rotation of 90°, 180°, and 270°

Of the 1568 possible placement of two rooks that do not obstruct each other, there are several possible positions that are the result of rotation or reflection of other configurations. With this in mind, the analysis can be carried out more efficiently by grouping all these possibilities in a symmetry.

This grouping based on symmetry serves to strategically analyze the strategy of the rook attack that is able to generate an equal pressure space against the opponent's king.

Looking at the configuration from a symmetrical point of view not only reduces the number of possible rook positions that need to be analyzed but can help understand the most effective rook placement patterns. By grouping the possible placement of strategically equal rook, the analysis can be focused on any one of the representatives without reviewing all the possibilities separately.

Fig 2.2 Placement of the position of the rook is symmetrically equivalent

F. Checkmate Teory and Matting Patterns

Checkmate is a condition in chess when the king is under direct threat and has no legal moves to avoid an attack. In the double rook attack strategy, checkmate occurs when two rooks coordinate with each other to create a situation where the opposing king is trapped and unable to escape.

Checkmate by using two rooks as an attack strategy has a distinctive pattern that often appears in the game. Some typical patterns include the following:

1. Back Rank Mate
   A classic checkmate that occurs when two rooks take control of the last row and column, as well as trap the opposing king who can't escape because he's trapped by his own pawn.
2. Sandwich Mate
   When one rook gives a check, while the other rook closes the escape route and forms pressure from both sides.
3. Cross Fire Mate
   When two rooks pose a threat from different directions and form a deadly cut-off point around the king.
4. Progressive Mate
   A checkmate pattern built in stages, the two rooks move in coordination to narrow the space of movement of the opposing king.

All of these theories can support the analysis of this research and will be the main focus in this study, so the discussion will focus more on attack strategies that use a combination of two rook to get checkmates and capture the opponent's king effectively.

III. Case Studies and Strategy Analysis

After knowing that there are 1568 possible positions of two rooks on the 8×8 chessboard, this study will analyze several cases of attack strategies that aim to control the area until they get checkmates and defense strategies from enemy attacks that only use two rooks to restore the situation. In chess, there are a variety of unique strategies that are effectively used to control the game, but this analysis will only discuss attack strategies that rely on the coordination of 2 rooks to control the game until you get a win.

1. Attack strategy

   Rooks are chess pieces that can be used for effective attacks, as the range of placing the rook is very wide as long as there are no pieces blocking its path. The strategies that will be analyzed in this case study are as follows:

   a. Classic Back Rank Mate

      In chess, a back rank mate is a checkmating pattern that happens when a player delivers checkmate by attacking the back rank of their opponent (first rank for white, eighth rank for black). For this checkmate to be possible, the squares in front of the kink must be blocked by its own pieces.

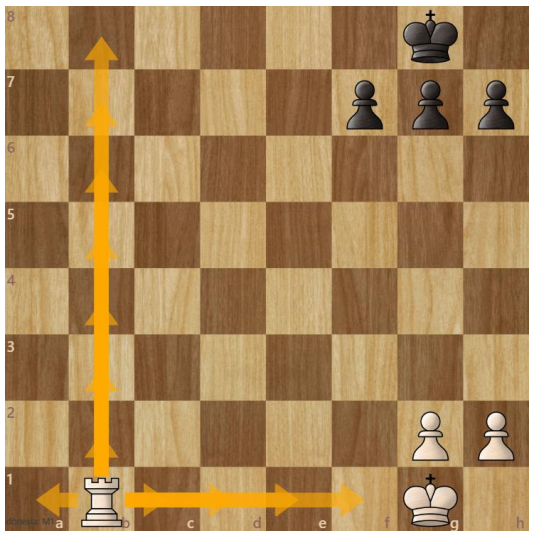

Figure 3.1 Analysis of the movement of the rook
(Taken from chess.com)

1. Starting position
   - White rook: b1
   - White king: g1
   - White pawn: g2, h2
   - Black king: g8
   - Black pawn: f7, g7, h7

2. Attack combinations that rooks can perform
   The white rook is in the position of b1 which can move horizontally or vertically anywhere, but the rook cannot move to g1 because there is a white king who is blocking it, so the rook also cannot move to h1. The possible movements of the rook can be done as follows:

   a. Vertical movement of the rook

   $$C(8,1) = \frac{8!}{7!\,(8-1)!} = \frac{8!}{7!} = 8\ positions$$

   Because the rook already occupies one square on the chessboard, then from 8 possibilities the movement of the rook will be reduced by 1 to reduce the square that has been filled by itself.

   $$movement\ of\ the\ rook = 8 - 1 = 7$$

   Because there are other pieces that block the path of the rook, namely the white king in g1, the rook also cannot go to h1, so it will be reduced by 2 to reduce the possibility of the position of the rook movement on the tile that has been filled with other pieces.

   $$movement\ of\ the\ rook = 7 - 1 - 1 = 5$$

   b. Horizontal movement of the rook
   On the horizontal path, there are no other pieces that can block the movement of the rook, so the possible movements that will be obtained are:

   $$C(8,1) = \frac{8!}{7!\,(8-1)!} = \frac{8!}{7!} = 8\ positions$$

   Because the rook already occupies one square on the chessboard, then from 8 possibilities the movement of the rook will be reduced by 1 to reduce the square that has been filled by itself.

   $$movement\ of\ the\ rook = 8 - 1 = 7$$

   c. Total possible movement of the rook

   $$movement\ of\ the\ rook = 7 + 5 = 12\ positions$$

3. Analysis of the movement of the rook to get the checkmate
   All possible movements of the white rook position are 12 tiles, taking into account all the tile positions that the white rook will effectively use to get checkmates and control the opponent's area until winning the match. Analyze each movement of the rook as follows:

   a. Move to a1
      If the white rook moves to a1, then the white rook does not move effectively to fully control the area of the opponent's chessboard.
   b. Move to c1
      If the white rook moves to c1, then the white rook does not move effectively to fully control the area of the opponent's chessboard.
   c. Move to d1
      If the white rook moves to d1, then the white rook does not move effectively to fully control the opponent's chessboard area.
   d. Move to e1
      If the white rook moves to e1, then the white rook does not move effectively to fully control the opponent's chessboard area.
   e. Move to f1
      If the white rook moves to f1, then the white rook does not move effectively to fully control the opponent's chessboard area.
   f. Move to b2
      If the white rook moves to b2, then the white rook does not move effectively to fully control the opponent's chess board area.
   g. Move to b3
      If the white rook moves to b3, then the white rook does not move effectively to fully control the opponent's chessboard area.
   h. Move to b4
      If the white rook moves to b4, then the white rook does not move effectively to fully control the opponent's chessboard area.
   i. Move to b5
      If the white rook moves to b5, then the white rook does not move effectively to fully control the opponent's chessboard area.

j. Move to b6

If the white rook moves to b6, then the white rook does not move effectively to fully control the opponent's chess board area.

k. Move to b7

If the white rook moves to b7, then the white rook can control horizontal are 7 where there are three opposing pawns on f7, g7, h7, so that the white rook can get one of the three opposing pawns. However, these movements are not effective enough to fully control the opponent's movement area, so they are not suitable for choice as the main attacker's movement strategy.

l. Move to b8

If the white rook moves to b8, then the white rook can fully control the opponent's king's area, so the opposing king can't escape anywhere. This movement becomes a very effective attacking option to use and win matches.

All of the options for moving the rook attack position from this analysis have their own uniqueness, but out of all the options there is only one attack option that can be used for an effective attack, which is to move to b8.

b. Sandwich Mate

Sandwich mate is a checkmate strategy that occurs when the king is trapped or pinched by his own piece. All chess players use other pieces to create defense for their king, but if too many pieces are used around the king it will result in the king being pinched by his own piece and the opponent can get a checkmate easily.

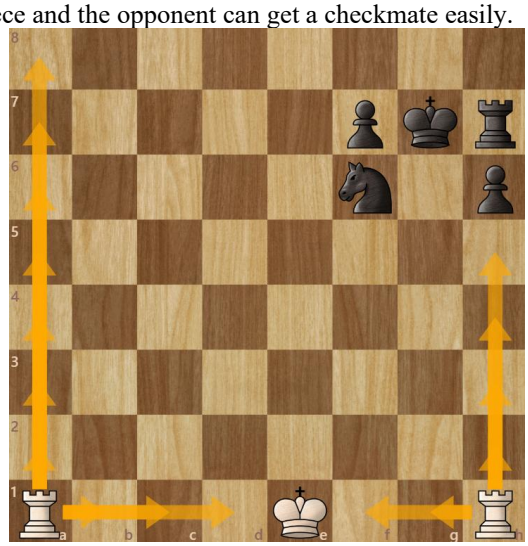

Figure 3.2 Analysis of the movement of the rook (Taken from chess.com)

1. Starting position
   - White rook: a1, h1
   - White king: e1
   - Black horse: f6
   - Black pawn: f7, h6
   - Black rook: h7
   - Black king: g7

2. Attack combinations that rooks can perform

In this sandwich mate analysis, the white player seems to have no hope of winning the match, because there are only two rooks left that can be used. However, the situation is not like that, white players are able to use the opportunity to do a sandwich mate strategy to win the match and get a checkmate with just 3 moves.

a. Horizontal movement of the rooks

White players can only use their two rooks to make moves. All possible position movements in this sandwich mate analysis to attack are as follows:

1. Horizontal movement of white rook a1

$$C(8,1) = \frac{8!}{7!(8-1)!} = \frac{8!}{7!} = 8\ positions$$

Because the horizontal movement of the white rook a1 is blocked by the white king on e1, so the white rook a1 cannot move to tiles e1, f1, g1, and h1. From the 8 results, the possibility of placing the position of the white rook movement a1 will be reduced by 1 for the plot where the white rook is placed and will be subtracted by 4 for the plot that is blocked by other pieces. All the possible horizontal movements of the white rook a1 are as follows:

$$movement\ of\ the\ rook = 8 - 1 - 4 = 3$$

The total horizontal movement of the white rook a1 is 3 tiles.

2. Horizontal movement of white rook h1

$$C(8,1) = \frac{8!}{7!(8-1)!} = \frac{8!}{7!} = 8\ positions$$

Because the movement of the white rook h1 is blocked by the king on e1, so the white rook h1 cannot move to tiles e1, d1, c1, b1, and a1. From the 8 possible placement of the white rook movement h1 will be reduced by 1 for the plot where the white rook is placed and will be subtracted by 5 for the plot that is blocked by other pieces. All the possible horizontal movements of the white rook h1 are as follows:

$$movement\ of\ the\ rook = 8 - 1 - 5 = 2$$

The total horizontal movement of the white rook h1 is 2 tiles.

b. Vertical movement of the rooks

White players can use their two rooks to move vertically anywhere in order to control the chessboard area, so they can turn the tide to win the game. Analyze all possible vertical movements of the two white rooks as follows:

1. Vertical movement of white rook a1

$$C(8,1) = \frac{8!}{7!(8-1)!} = \frac{8!}{7!} = 8\ positions$$

The white rook a1 can move freely anywhere vertically. All the possible movements of the white rook a1 are 8 tiles, but will be subtracted by 1 for the tiles that have already laid themselves out. All valid a1 white rook vertical movement possibilities are as follows:

$$movement\ of\ the\ rook = 8 - 1 = 7$$

2. Vertical movement of white rook h1

$$C(8,1) = \frac{8!}{7!(8-1)!} = \frac{8!}{7!} = 8\ positions$$

The white rook h1 has 8 possibilities of moving vertically if there are no other pieces blocking it. However, there are black pawns and black rooks that block the vertical movement path of the white rook a1, then the movement will be reduced by 3, because it cannot move to h6, h7, and h8, and will be reduced by 1 for the tiles that have been occupied by the white rook h1.

$$movement\ of\ the\ rook = 8 - 1 - 3 = 4$$

c. Analysis of the movement of the rook to get the checkmate

The analysis of all the possible coordinated movements of the two rooks to get a checkmate and win the match from the sandwich strategy is very easy, if the opponent places another piece around the king, so that the opposing king is pinched by the other piece, then we will take advantage of the opportunity to attack the narrow space of the opponent's area and control the match. The analysis of the coordination movement of the two rooks is as follows:

1. White rook h1 moves horizontally to g1

   White players can check the opponent's king first, which will make the game area controllable and force the opposing king to retreat to the f8, g8, or h8 tiles.

2. White rook h1 moves vertically to a8

   Since the opposing king is already pinned by his own piece in area 8, the white rook a1 can move to square a8 to attack the opponent's king and can have more control over the opponent's area. The opponent will automatically place a black horse from f6 to e8 to help defend against the black king.

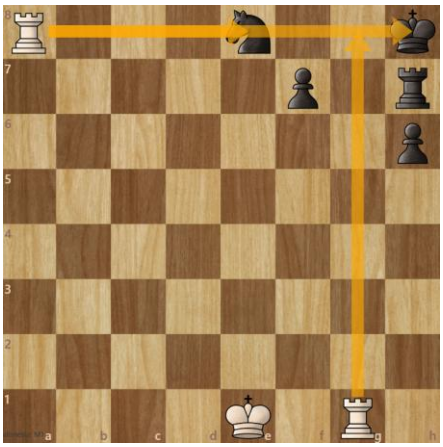

Figure 3.3 Analysis of the movement of the rook (Taken from chess.com)

3. White rook moves horizontally to e8

   In this condition, the g-line is already controlled by the white rook g1 and the white rook a8 is able to move the dark horse to e8 freely, so that it will cause the checkmate and the white player to win the match.

c. Cross Fire Mate

Cross fire mate is a cross-checkmate that occurs when the player gives a check to the opposing king while also giving a check to another opponent's piece that attacks the king, so that the attacking chess piece cannot move to block the checkmate.

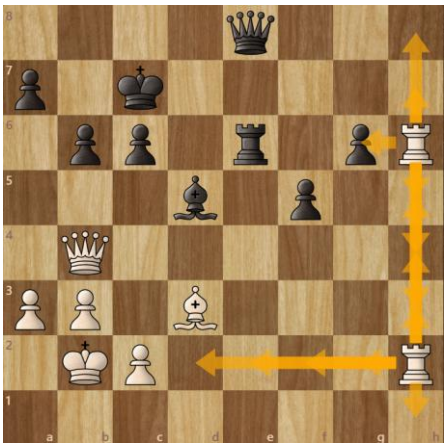

Figure 3.4 Analysis of the movement of the rook (Taken from chess.com)

1. Starting Position
   - White pawns: a3, b3, c2
   - White king: b4
   - White bishop: d3
   - White rooks: h2, h6
   - White queen: c3
   - Black rooks: e6
   - Black pawns: a7, b6, c6, g6, f5
   - Black king: b7

- Black bishop: d5
- Black queen: e8

2. Attack combinations that rooks can perform

In the cross-fire mate analysis, two rooks coordinate with each other to cross attack the opposing king assisted by another piece to get a checkmate.

a. Horizontal movement of the rooks

The white player gets a turn to place the pieces first, in the cross fire mate analysis of two rooks, the white player can find all the possible positions of the effective attack movement of the rook, Analyze all the possible movements of the two rooks as follows:

1. Horizontal movement of the white rook h2

$$C(8,1) = \frac{8!}{7!\,(8-1)!} = \frac{8!}{7!} = 8\ positions$$

In this analysis, the horizontal movement of the white rook h2 is blocked by the pawn on c2 which results in the horizontal movement of the white rook h2 cannot be placed on tiles c2, b2, and a2, then all possibilities will be reduced by 3, and will be reduced by 1 for the white rook that has been placed on h2. All possibilities for horizontal positioning of the h2 white rook are as follows:

$$movement\ of\ the\ rook = 8 - 1 - 3 = 4$$

All possibilities of placing the white rook position h2 horizontally are 4 ways.

2. Horizontal movement of white rook h6

$$C(8,1) = \frac{8!}{7!\,(8-1)!} = \frac{8!}{7!} = 8\ positions$$

In this analysis, the horizontal movement of the white rook h6 is blocked by the opponent's black pawn, so the rook can attack the opponent's black pawn on g6, so the h6 rook cannot be placed on a6, b6, c6, d6, e6, f6. However, from the 8 possible positions of the white rook movement h6 will be reduced by 6 tiles that cannot be placed by the white rook h6, because it is blocked by the black pawn of g6, and will be reduced by 1 for the white rook that has been placed on h6.

$$movement\ of\ the\ rook = 8 - 6 - 1 = 1$$

All the possibilities of placing the white rook h6 horizontally have only 1 way which is to go to g6 and attack the enemy's black pawn.

b. Vertical movement of the rooks

In this cross fire mate analysis, the vertical movement of two rooks obstructs each other, so one of the rook is needed to move freely to another column or row so that these two white rooks do not block each other. However, I will analyze all the possibilities of the initial positions of these two rooks when they block each other first.

1. Vertical movement of white rook h2

$$C(8,1) = \frac{8!}{7!\,(8-1)!} = \frac{8!}{7!} = 8\ positions$$

White players can move the h2 white rook vertically anywhere if there are no other pieces in its way. However, these two white rooks were in the same column, thus blocking their path of movement from each other. The white rook h2 is not blocked by the white rook h6 which results in the white rook h2 cannot be placed on h6, h7, and h8. Since the white rook h2 cannot be placed in the three tiles, then all the possible movements of the rook h2 will be reduced by 3 and will be subtracted by 1, because the white rook has already been placed in the square h2.

$$movement\ of\ the\ rook = 8 - 1 - 3 = 4$$

All possibilities of placing the white rook position h2 horizontally are 4 ways.

2. Vertical movement of white rook h6

$$C(8,1) = \frac{8!}{7!\,(8-1)!} = \frac{8!}{7!} = 8\ positions$$

Because there is a white rook h2 that blocks the vertical movement of the white rook h6, then from all possibilities of placing the white rook h6 will be reduced by 2, because the rook h6 cannot be placed on the h1 and h2 tiles, and will be subtracted by 1 for the white rook that has been placed on one of the chessboard tiles, namely h6. Analyze all possible movements of the h6 rook vertically as follows:

$$movement\ of\ the\ rook = 8 - 1 - 2 = 5$$

All possible placement of the white rook h6 vertically has 5 ways.

c. Analysis of the movement of the rook to get the checkmate

Effective analysis that will be used for crossfire mate strategy, so as to be able to win the match as follows:

1. White rook h6 moves to h7

From the arrangement of the opposing player's chess pieces that have weaknesses, then we will use this opportunity to check the opponent's king first by using the white rook h6 moves to h7. Since the opposing king is threatened, the black rook of e6 automatically moves to e7 to make a defense.

2. White queen moves from b4 to e7

   The opponent's black rook e7 is in the attack area of the white queen b4, so the queen can move from b4 to e7 to help the attack and get checks. The enemy black queen's side will attack from e8 to e7 to defend the king's area.
3. White rook h7 moves to e7

   The opponent's black queen in the e7 tile is within range of the attack of the white rook h7 which can be attacked so that it gets a check. Automatically the opposing king will move from c7 to b8, c8, or d8.
4. White rook h2 moves to h8

   The last move to win the match is to place the white rook h2 to h8, so the opponent's black king will be hit by the white rook h8 and the white rook e7 and will be hit by a checkmate.

IV. CONCLUSION

The combinatorial concept can be used to solve various possible chess piece movements, especially rooks, which is the main topic in this analysis. Using the combinatorial concept, various steps can be used to make decisions on the movement of the chess piece. Many beginner chess players feel that they do not have the talent to play chess, but with a lot of practice and calculating all the possible moves that will be used using the combinatorial concept, they will become a reliable chess player at some point. This combinatorial concept provides a starting foundation for chess players to learn more about the concept of combinatorics in chess well.

VIDEO LINK AT YOUTUBE

https://youtu.be/RfI-imqRE7I?si=0MogvWOgdT6LSPn7

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

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